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# Bi-Objective Reliability Based Optimization: An Application to Investment Analysis

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**Abstract**

We solve three bi-objective portfolio optimization problems with probabilistic constraints which are intuitive and practical. We derive the expressions of risks (i.e., expected loss) of a portfolio, under asymmetric (namely LINear EXponential (LINEX) and relative LINEX) loss functions, considering distribution of assets returns are Gumbel (example of Extreme Value Distribution (EVD)). We propose three bi-objective Reliability Portfolio Optimization Models (RPOMs), whereby in each of the model the first objective is related to minimization of an unknown threshold value such that the risk of the portfolio returns is less than or equal to this unknown threshold, by a known fixed level of reliability. The second part in each of the bi-objective optimization functions is associated with maximizing a certain different threshold such that the expected return of the portfolio, is more than or equal to the unknown threshold, by a predefined fixed but different reliability level. In the first RPOM, the returns are EVD while risk is LINEX loss based. In the second model the EVD returns are calculated relative to risk free interest rate, hence commensurate risk based on relative LINEX loss is utilized. The third model has the same form of expected return in its objective function as the first model, while the risk is based on Squared Error Loss (SEL). We test the efficacy of the models using data from the Indian stock market.

## Highlights

Inclusion of data uncertainties in reliability based financial optimization enable judicious planning of risk and return.

## Highlights

EVT and ARCH/GARCH modeling predicts better asset returns which are utilized in reliability based financial optimization.

## Highlights

Optimal choices of reliability indices help investors prudently decide the weights of different assets depending on his/her risk appetite.

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# Description of Models



**Bi-Objective Mean-Variance (MV) optimization problem**

$$max: \sum_{i=1}^{N} w_i \bar{r}_i \quad min: \sum_{i=1}^{N}\sum_{j=1}^{N} w_i w_j \hat{\sigma}_{i,j}$$
$$s.t: \sum_{i=1}^{N} w_i \bar{r}_i \geq r_P^*$$
$$\sum_{i=1}^{N}\sum_{j=1}^{N} w_i w_j \hat{\sigma}_{i,j} \leq \sigma_P^{2*}$$
$$\sum_{i=1}^{N} w_i = 1$$
$$0 \leq w_{i,min} \leq w_i \leq w_{i,max} \leq 1$$

**Bi-Objective Mean-CVaR optimization problem**

$$max: \sum_{i=1}^{N} w_i \bar{r}_i \quad min: \left\{\frac{1}{\alpha T}\sum_{t=1}^{T}\sum_{i=1}^{N}\left(r_{i,t} w_i - \gamma\right)^{+} + \gamma\right\}$$
$$s.t: \sum_{i=1}^{N} w_i \bar{r}_i \geq r_P^*$$
$$\left\{\frac{1}{\alpha T}\sum_{t=1}^{T}\sum_{i=1}^{N}\left(r_{i,t} w_i - \gamma\right)^{+} + \gamma\right\} \leq CVaR^*$$
$$\sum_{i=1}^{N} w_i = 1$$
$$0 \leq w_{i,min} \leq w_i \leq w_{i,max} \leq 1$$

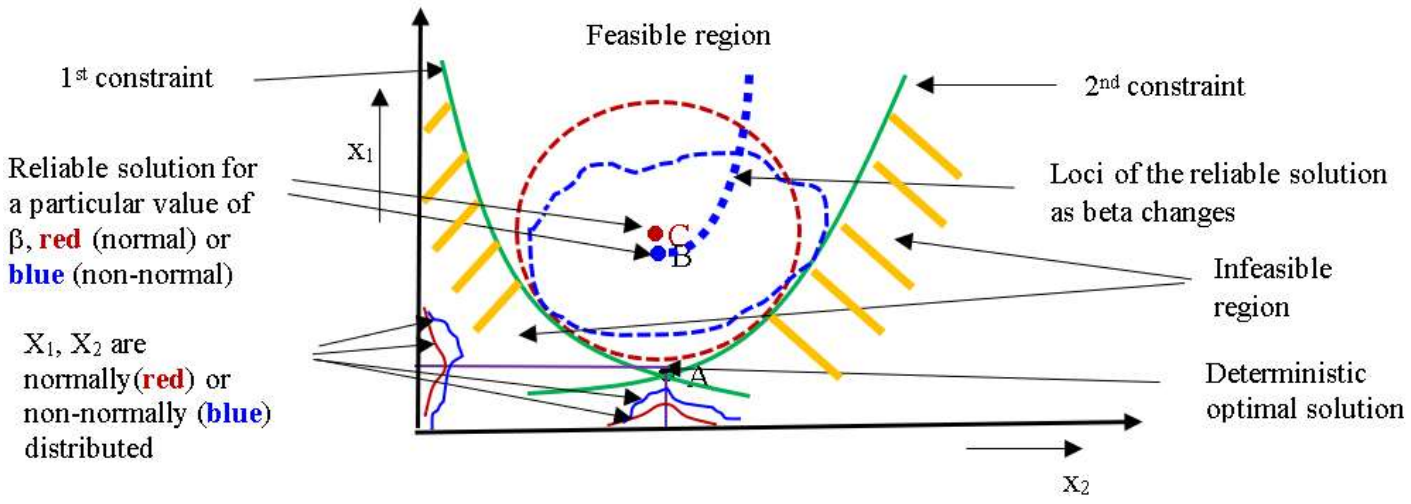
**Bi-Objective Mean-EVaR optimization problem**

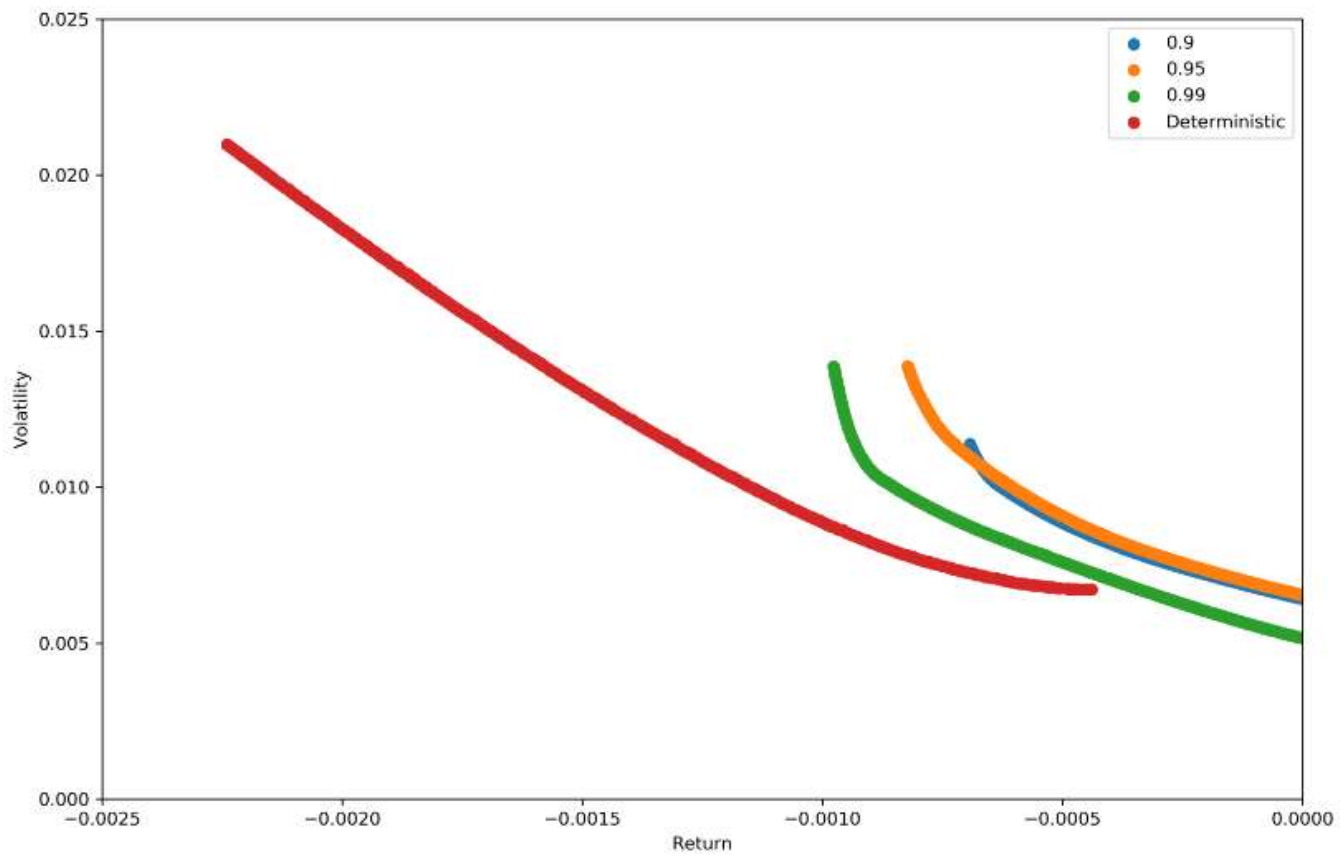
$$max: \sum_{i=1}^{N} w_i \bar{r}_i \quad min: \left\{\gamma \ln \sum_{t=1}^{T} \frac{1}{T} e^{-\gamma^{-1}\sum_{i=1}^{N} w_i \bar{r}_i}\right\}$$
$$s.t: \sum_{i=1}^{N} w_i \bar{r}_i \geq r_P^*$$
$$\left\{\gamma \ln \sum_{t=1}^{T} \frac{1}{T} e^{-\gamma^{-1}\sum_{i=1}^{N} w_i \bar{r}_i}\right\} \leq EVaR^*$$
$$\sum_{i=1}^{N} w_i = 1$$
$$0 \leq w_{i,min} \leq w_i \leq w_{i,max} \leq 1$$

# Description of Data Set

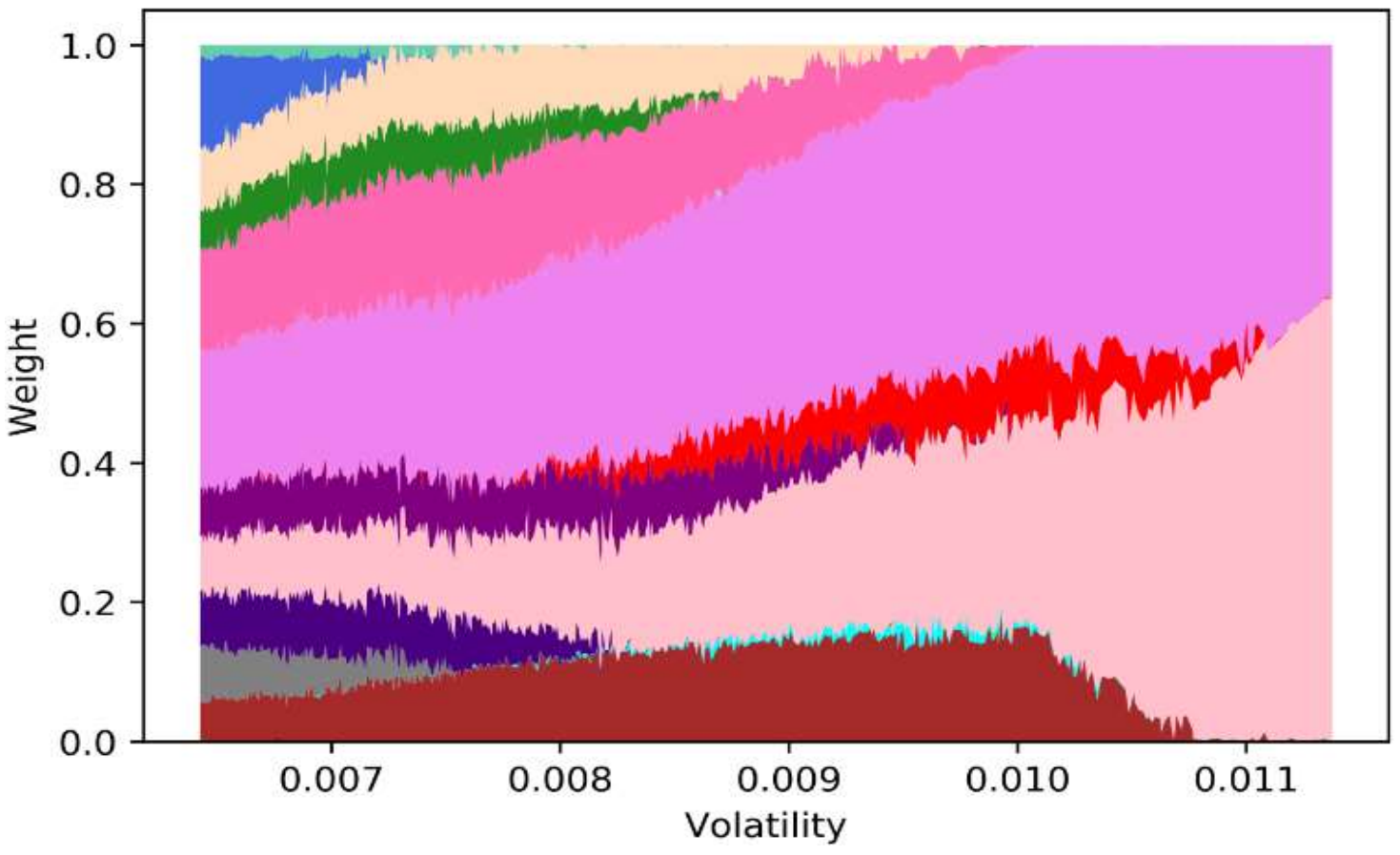
The dataset utilized for our research is BSE 30 index from Bombay Stock Exchange (BSE), INDIA <https://www.bseindia.com/> for the time frame (01-Jan-2014 to 01-Jan-2020). BSE30 index consists of the following stocks namely, Asian Paints (BSE: 500820), Axis Bank (BSE: 532215), Bajaj Auto (BSE: 532977), Bajaj Finance (BSE: 500034), Bharti Airtel (BSE: 532454), HCL Technologies (BSE: 532281), HDFC Bank (BSE: 500180), Hero MotoCorp (BSE: 500182), Hindustan Unilever (BSE: 500696), Housing Development Finance Corporation (BSE: 500010), ICICI Bank (BSE: 532174), IndusInd Bank (BSE: 532187), Infosys (BSE: 500209), ITC Ltd. (BSE: 500875), Kotak Mahindra Bank (BSE: 500247), Larsen & Toubro (BSE: 500510), Mahindra and Mahindra (BSE: 500520), Maruti Suzuki India (BSE: 532500), Nestle India (BSE: 500790), NTPC (BSE: 532555), Oil & Natural Gas Corporation (BSE: 500312), Power Grid Corporation of India (BSE: 532898), Reliance Industries (BSE: 500325), State Bank of India (BSE: 500112), Sun Pharmaceutical Industries (BSE: 524715), Tata Consultancy Services (BSE: 532540), Tata Steel (BSE: 500470), Tech Mahindra (BSE: 532755), Titan Company (BSE: 500114), Ultra Tech Cement(BSE: 532538).

| SNo. | BSE30 Script | BSE30 Ticker | Colour | Colour Name | RGB code | Hexa |
|---|---|---|---|---|---|---|
| 01 | Asian Paints | 500820 | | Purple | R (128), G (000), B (128) | #800080 |
| 02 | Axis Bank | 532215 | | Green | R (000), G (255), B (000) | #00FF00 |
| 03 | Bajaj Auto | 532977 | | Sunset | R (255), G (214), B (165) | #FFD6A5 |
| 04 | Bajaj Finance | 500034 | | Picton Blue | R (000), G (176), B (240) | #00B0F0 |
| 05 | Bharti Airtel | 532454 | | Bright Yellow | R (255), G (176), B (001) | #FFB001 |
| 06 | HCL Technologies | 532281 | | Rust | R (176), G (078), B (034) | #B04E22 |
| 07 | HDFC Bank | 500180 | | Black | R (000), G (000), B (000) | #000000 |
| 08 | Hero Motor Corp | 500182 | | Pearl Aqua | R (148), G (210), B(191) | #94D2BF |
| 09 | Hindustan Unilever | 500696 | | Falu Red | R (132), G (034), B (034) | #842222 |
| 10 | Housing Development Finance Corporation | 500010 | | Crimson Red | R (156), G (010), B (010) | #9C0A0A |
| 11 | ICICI Bank | 532174 | | Electric Cyan | R (000), G (255), B(255) | #00FFFF |
| 12 | IndusInd Bank | 532187 | | Red | R (255), G (000), B (000) | #FF0000 |
| 13 | Infosys | 500209 | | Trolley Grey | R (128), G (128), B(128) | #808080 |
| 14 | ITC | 500875 | | Grape | R (112), G (048), B(160) | #7030A0 |
| 15 | Kotak Mahindra Bank | 500247 | | Phthalo Blue | R (025), G (017), B(129) | #191181 |
| 16 | Larsen & Toubro | 500510 | | Pale goldenrod | R (242), G (237), B(166) | #F2EDA6 |
| 17 | Mahindra and Mahindra | 500520 | | White | R (255), G (255), B(255) | #FFFFFF |
| 18 | Maruti Suzuki India | 532500 | | Light Pink | R (240), G (168), B(180) | #F0A8B4 |
| 19 | Nestlé India | 500790 | | Flamingo Pink | R (235), G (114), B(244) | #EB72F4 |
| 20 | NTPC | 532555 | | French Pink | R (255), G (102), B(153) | #FF6699 |
| 21 | Oil & Natural Gas Corporation | 500312 | | Ultramarine Blue | R (000), G (102), B(255) | #0066FF |
| 22 | Power Grid Corporation of India | 532898 | | Vivid Violet | R (138), G (040), B(168) | #8A28A8 |
| 23 | Reliance Industries | 500325 | | Blue | R (000), G (000), B(242) | #0000F2 |
| 24 | State Bank of India | 500112 | | Ultramarine | R (000), G (000), B(153) | #000099 |
| 25 | Sun Pharmaceutical Industries | 524715 | | Irish Green | R (051), G (153), B(051) | #339933 |
| 26 | Tata Consultancy Services | 532540 | | Office Green | R (000), G (128), B (000) | #008000 |
| 27 | Tata Steel | 500470 | | Safety Orange | R (255), G (134), B(051) | #FF8633 |
| 28 | Tech Mahindra | 532755 | | Ruby Red | R (165), G (000), B(033) | #A50021 |
| 29 | Titan Company | 500114 | | Cadmium Yellow | R (252), G (246), B(010) | #FCF60A |
| 30 | Ultra Tech Cement | 532538 | | Aquamarine | R (153), G (255), B(209) | #99FFD1 |




Pareto Frontier for Model I (with reliabilities on returns and variances) considering deterministic and probabilistic $(\beta_1, \beta_2 = 0.90; \beta_1\beta_2, = 0.95; \beta_1\beta_2 = 0.99)$ constraints




Weight distribution for Model I (with reliability on returns and variances) considering $\beta_1, \beta_2 = 0.90$